MICROECONOMICS II

ECON 312

Game Theory

Lecture 4: Week 4 & 5

Dr. Kwame Agyire-Tettey

Recommended Textbook

Hal R. Varian – 8Th Edition

Chapter 28

Game Theory

- Game theory is used to model strategic behaviour by agents who understand that their actions affect the actions of other agents.
- It involves strategic interactions of agents
- Indeed, most economic behaviour can be viewed as special cases of game theory, and a sound understanding of game theory is a necessary component of any economist's set of analytical tools.

Some Applications of Game Theory

- The study of oligopolies (industries containing only a few firms)
- The study of cartels; e.g. OPEC
- The study of externalities; e.g. using a common resource such as a fishery.
- Businesses competing in a market.
- Bargaining and workings of markets
- The study of military strategies.
- Diplomats negotiating a treaty.
- Gamblers betting in a card game

What is a Game?

- A **game** consists of
  - a set of **players**
  - a set of **strategies** for each player, which consist a sequence of **moves**.
  - optimal strategy is a sequence of moves that results in your best **outcome**.
  - the **payoffs** to each player for every possible list of strategy choices by the players.

Types of Games

- There are two fundamental types of games: sequential and simultaneous.
- In **sequential games**, the players must alternate moves.
- In **simultaneous games**, the players can act at the same time.
- These types are distinguished because they require different analytical approaches.

Two-Player Games

- A game with just two players is a **two-player game**.
- We will study only games in which there are two players, each of whom can choose between only **two strategies**.

An Example of a Two-Player Game

- The players are called A and B.
- Player A has two strategies, called **"Up" and "Down"**.
- Player B has two strategies, called **"Left" and "Right"**.
- The table showing the payoffs to both players for each of the four possible strategy combinations is the game's **payoff matrix**.

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

This is the game's payoff matrix.

Player A's payoff is shown first.
Player B's payoff is shown second.

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

A "play" of the game is a pair, such as (U,R) where the 1st element is the strategy chosen by Player A and the 2nd is the strategy chosen by Player B.

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

This is the game's payoff matrix.

E.g. if A plays Up and B plays Right then A's payoff is 1 and B's payoff is 8.

An Example of a Two-Player Game

		Player B L	Player B R
Player A	U	(3,9)	(1,8)
Player A	D	(0,0)	(2,1)

This is the game's payoff matrix.

And if A plays Down and B plays Right then A's payoff is 2 and B's payoff is 1.

An Example of a Two-Player Game

Player B

		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

What plays are we likely to see for this game?

An Example of a Two-Player Game

Player B

Player A

	L	R
U	(3,9)	(1,8)
D	(0,0)	(2,1)

Is (U,R) a likely play?

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (U,R) a likely play?

If B plays Right then A's best reply is Down since this improves A's payoff from 1 to 2. So (U,R) is not a likely play.

An Example of a Two-Player Game

		Player B L	Player B R
Player A	U	(3,9)	(1,8)
Player A	D	(0,0)	(2,1)

Is (D,R) a likely play?

An Example of a Two-Player Game

		Player B L	Player B R
Player A	U	(3,9)	(1,8)
Player A	D	(0,0)	(2,1)

Is (D,R) a likely play?

If B plays Right then A's best reply is Down.

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (D,R) a likely play?

If B plays Right then A's best reply is Down.
If A plays Down then B's best reply is Right.
So (D,R) is a likely play.

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (D,L) a likely play?

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (D,L) a likely play?

If A plays Down then B's best reply is Right, so (D,L) is not a likely play.

An Example of a Two-Player Game

Player B

		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (U,L) a likely play?

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (U,L) a likely play?

If A plays Up then B's best reply is Left.

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (U,L) a likely play?

If A plays Up then B's best reply is Left.
If B plays Left then A's best reply is Up.
So (U,L) is a likely play.

Nash Equilibrium

- A play of the game where each strategy is a best reply to the other is a **Nash equilibrium**.
- the outcome of a game is a Nash equilibrium if the strategy followed by each player maximises that player's payoff given the strategies followed by all the other players.
- Our example has two Nash equilibria; (U,L) and (D,R).

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

(U,L) and (D,R) are both Nash equilibria for the game.

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

(U,L) & (D,R) are both Nash equilibria for the game

But which will we see? Notice that (U,L) is preferred to (D,R) by both players. Must we then see (U,L) only?

The Prisoner's Dilemma

- **To see if *Pareto-preferred* outcomes must be what we see in the play of a game, consider a famous second example of a two-player game called the Prisoner's Dilemma.**

The Prisoner's Dilemma

Bonnie and Clyde are arrested.

The police can prove illegal weapon possession.

The confession of the accomplice would be a proof of homicide.

The two are questioned separately

promise of a pardon if accuse the accomplice

if both accuse they get joint murder (10 years)

if none accuses both get illegal weapon possession (5 years)

if one accuses and the other doesn't (30 years and leniency 1 year)

The Prisoner's Dilemma

		Clyde: S	Clyde: C
Bonnie	**S**	**(-5,-5)**	**(-30,-1)**
	C	**(-1,-30)**	**(-10,-10)**

What plays are we likely to see for this game?

The Prisoner's Dilemma

		Clyde S	Clyde C
Bonnie	S	(-5,-5)	(-30,-1)
Bonnie	C	(-1,-30)	(-10,-10)

If Bonnie plays Silence then Clyde's best reply is Confess.

The Prisoner's Dilemma

		Clyde S	Clyde C
Bonnie	S	(-5,-5)	(-30,-1)
Bonnie	C	(-1,-30)	(-10,-10)

If Bonnie plays Silence then Clyde's best reply is Confess.
If Bonnie plays Confess then Clyde's best reply is Confess.

The Prisoner's Dilemma

		Clyde S	Clyde C
Bonnie	S	(-5,-5)	(-30,-1)
	C	(-1,-30)	(-10,-10)

So no matter what Bonnie plays, Clyde's best reply is always Confess.
Confess is a dominant strategy for Clyde.

The Prisoner's Dilemma

		Clyde S	Clyde C
Bonnie	S	(-5,-5)	(-30,-1)
	C	(-1,-30)	(-10,-10)

Similarly, no matter what Clyde plays, Bonnie's best reply is always Confess. Confess is a dominant strategy for Bonnie also.

The Prisoner's Dilemma

		Clyde S	Clyde C
Bonnie	S	(-5,-5)	(-30,-1)
Bonnie	C	(-1,-30)	(-10,-10)

So the only Nash equilibrium for this game is (C,C), even though (S,S) gives both Bonnie and Clyde better payoffs. The only Nash equilibrium is inefficient.

Who Plays When?

- In both examples the players chose their strategies simultaneously.
- Such games are **simultaneous play games**.

Who Plays When?

- But there are games in which one player plays before another player.
- Such games are **sequential play games**.
- The player who plays first is the **leader**. The player who plays second is the **follower**.

A Sequential Game Example

- Sometimes a game has more than one Nash equilibrium and it is hard to say which is more likely to occur.
- When such a game is sequential it is sometimes possible to argue that one of the Nash equilibria is more likely to occur than the other.

A Sequential Game Example

		Player B L	Player B R
Player A	U	(3,9)	(1,8)
Player A	D	(0,0)	(2,1)

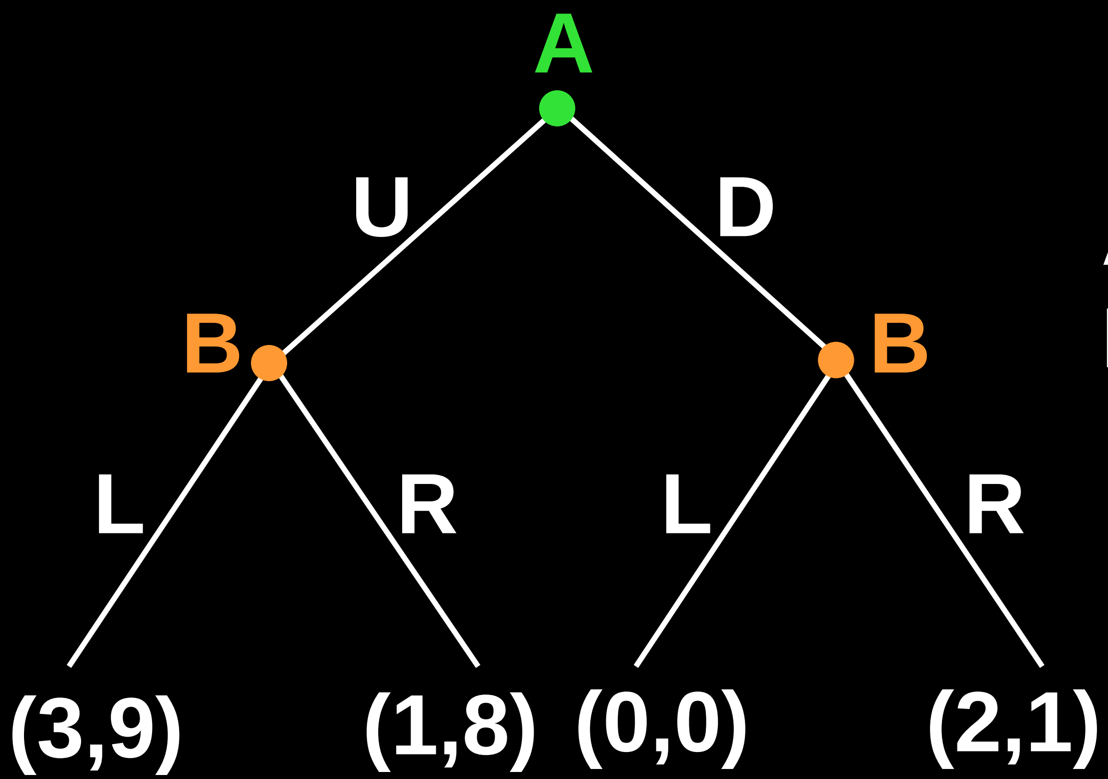
(U,L) and (D,R) are both Nash equilibria when this game is played simultaneously and we have no way of deciding which equilibrium is more likely to occur.

A Sequential Game Example

		Player B L	Player B R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

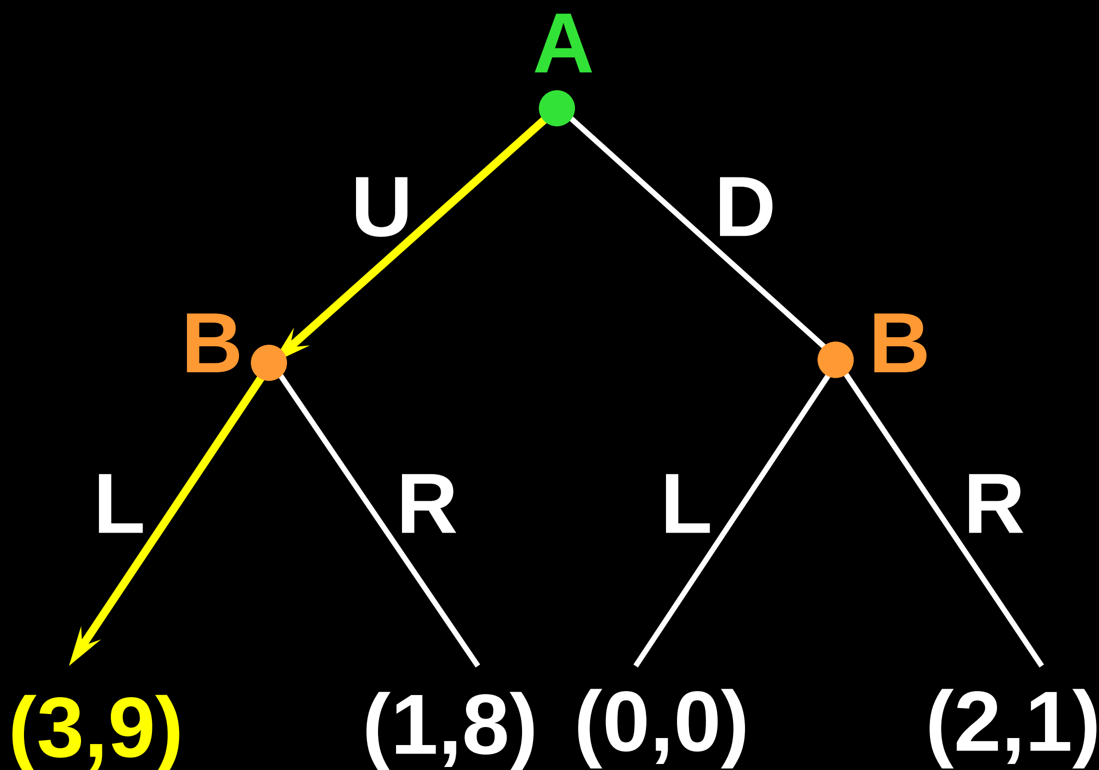
Suppose instead that the game is played sequentially, with A leading and B following. We can rewrite the game in its extensive form.

A Sequential Game Example



A plays first.
B plays second.

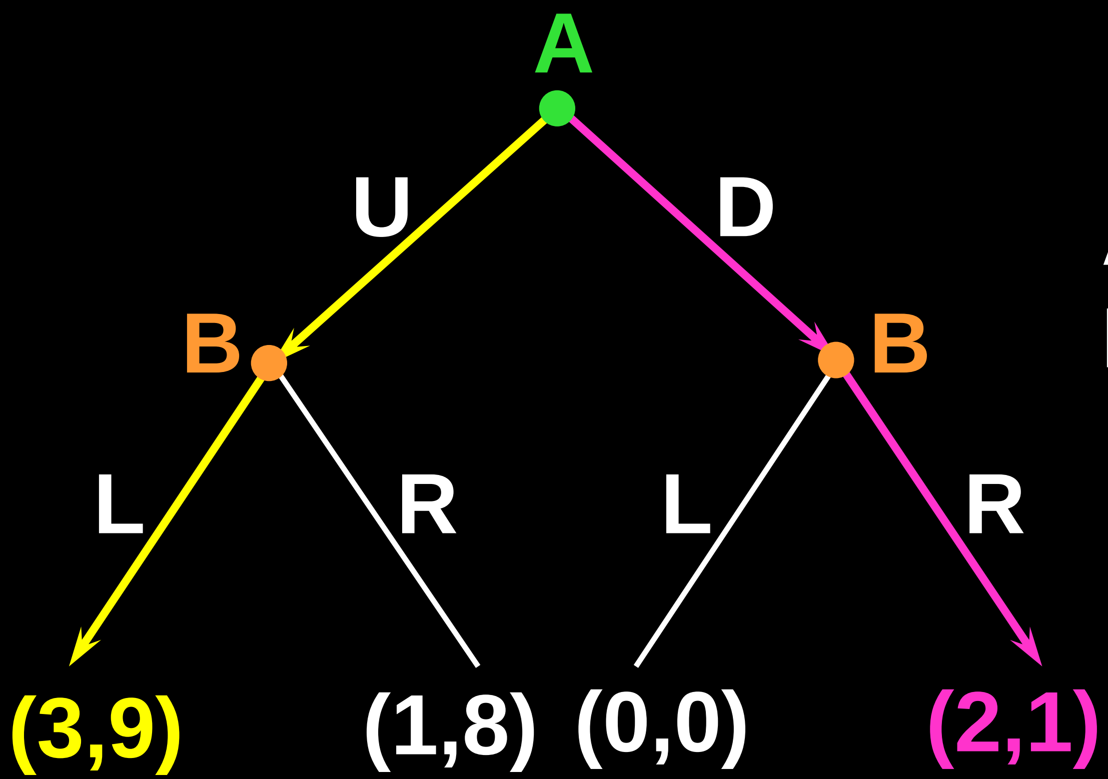
A Sequential Game Example



A plays first.
B plays second.

(U,L) is a Nash equilibrium.

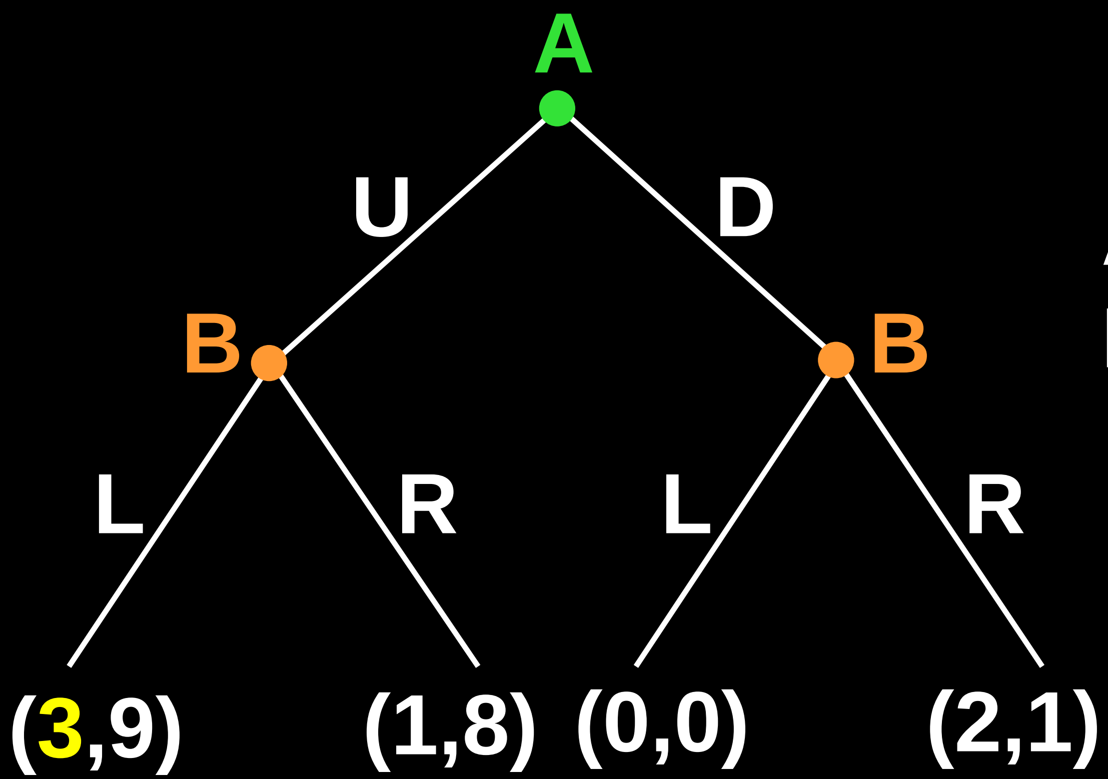
A Sequential Game Example



A plays first.
B plays second.

(U,L) is a Nash equilibrium.
(D,R) is a Nash equilibrium.
Which is more likely to occur?

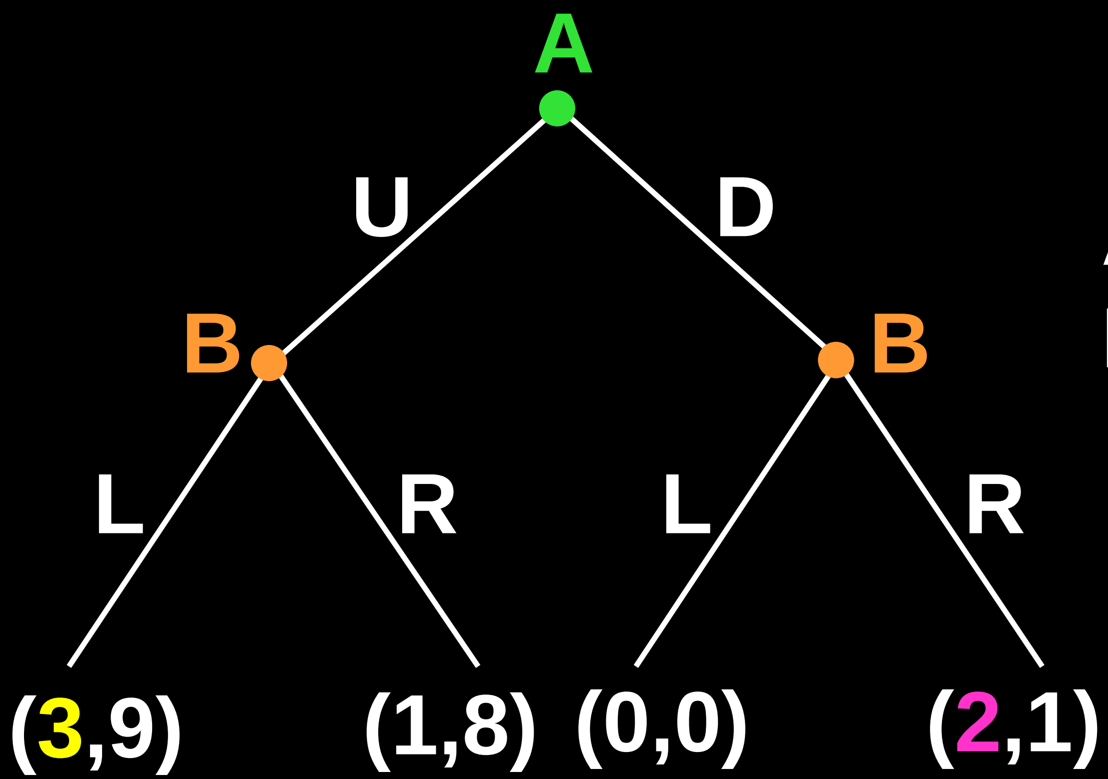
A Sequential Game Example



A plays first.
B plays second.

If A plays U then B plays L; A gets 3.

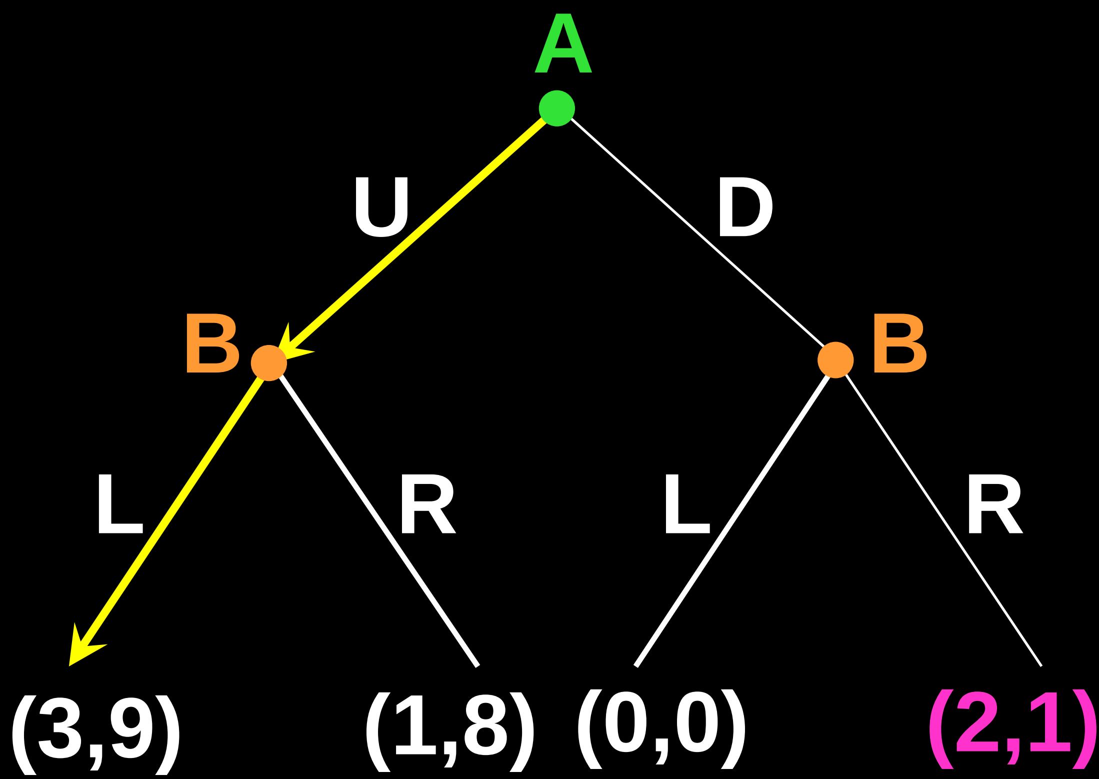
A Sequential Game Example



A plays first.
B plays second.

If A plays U then B plays L; A gets 3.
If A plays D then B plays R; A gets 2.

A Sequential Game Example



A plays first.
B plays second.

If A plays U then B plays L; A gets 3.
If A plays D then B plays R; A gets 2.
So (U,L) is the likely Nash equilibrium.

Pure Strategies

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

This is our original example once more. Suppose again that play is simultaneous. We discovered that the game has two Nash equilibria; (U,L) and (D,R).

Pure Strategies

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Player A's has been thought of as choosing to play either U or D, but no combination of both; that is, as playing purely U or D.

U and D are Player A's pure strategies.

Pure Strategies

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Similarly, L and R are Player B's pure strategies.

Pure Strategies

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Consequently, (U,L) and (D,R) are pure strategy Nash equilibria. Must every game have at least one pure strategy Nash equilibrium?

Pure Strategies

		Player B L	Player B R
Player A	U	(1,2)	(0,4)
	D	(0,5)	(3,2)

Here is a new game. Are there any pure strategy Nash equilibria?

Pure Strategies

		Player B: L	Player B: R
Player A	U	(1,2)	(0,4)
	D	(0,5)	(3,2)

Is (U,L) a Nash equilibrium?

Pure Strategies

Player B

		L	R
Player A	U	(1,2)	(0,4)
	D	(0,5)	(3,2)

Is (U,L) a Nash equilibrium? No.

Is (U,R) a Nash equilibrium?

Pure Strategies

		Player B	
		L	R
Player A	U	(1,2)	(0,4)
	D	(0,5)	(3,2)

Is (U,L) a Nash equilibrium? No.
Is (U,R) a Nash equilibrium? No.
Is (D,L) a Nash equilibrium?

Pure Strategies

		Player B	
		L	R
Player A	U	(1,2)	(0,4)
	D	(0,5)	(3,2)

Is (U,L) a Nash equilibrium? No.
Is (U,R) a Nash equilibrium? No.
Is (D,L) a Nash equilibrium? No.
Is (D,R) a Nash equilibrium?

Pure Strategies

		Player B L	Player B R
Player A	U	(1,2)	(0,4)
Player A	D	(0,5)	(3,2)

Is (U,L) a Nash equilibrium? No.
Is (U,R) a Nash equilibrium? No.
Is (D,L) a Nash equilibrium? No.
Is (D,R) a Nash equilibrium? No.

Pure Strategies

		Player B L	Player B R
Player A	U	(1,2)	(0,4)
Player A	D	(0,5)	(3,2)

So the game has no Nash equilibria in pure strategies. Even so, the game does have a Nash equilibrium, but in mixed strategies.

Mixed Strategies

- **Instead of playing purely Up or Down, Player A selects a probability distribution (π_U,1-π_U), meaning that with probability π_U Player A will play Up and with probability 1-π_U will play Down.**
- **Player A is mixing over the pure strategies Up and Down.**
- **The probability distribution (π_U,1-π_U) is a mixed strategy for Player A.**

Mixed Strategies

- **Similarly, Player B selects a probability distribution (π_L,1-π_L), meaning that with probability π_L Player B will play Left and with probability 1-π_L will play Right.**
- **Player B is mixing over the pure strategies Left and Right.**
- **The probability distribution (π_L,1-π_L) is a mixed strategy for Player B.**

Mixed Strategies

		Player B	
		L	R
Player A	U	(1,2)	(0,4)
	D	(0,5)	(3,2)

This game has no pure strategy Nash equilibria but it does have a Nash equilibrium in mixed strategies. How is it computed?

Mixed Strategies

		Player B	
		L, π_L	R, $1-\pi_L$
Player A	U, π_U	(1,2)	(0,4)
	D, $1-\pi_U$	(0,5)	(3,2)

Mixed Strategies

		Player B	
		L, π_L	R, 1-π_L
Player A	U, π_U	(1,2)	(0,4)
	D, 1-π_U	(0,5)	(3,2)

If B plays Left her expected payoff is

$$2\pi_U + 5(1 - \pi_U)$$

Mixed Strategies

Player B

		L, π_L	$R, 1-\pi_L$
Player A	U, π_U	(1,2)	(0,4)
	$D, 1-\pi_U$	(0,5)	(3,2)

If B plays Left her expected payoff is

$$2\pi_U + 5(1-\pi_U).$$

If B plays Right her expected payoff is

$$4\pi_U + 2(1-\pi_U).$$

Mixed Strategies

Player B

		L, π_L	$R, 1-\pi_L$
Player A	U, π_U	(1,2)	(0,4)
	$D, 1-\pi_U$	(0,5)	(3,2)

If $2\pi_U + 5(1-\pi_U) > 4\pi_U + 2(1-\pi_U)$ then B would play only Left. But there are no Nash equilibria in which B plays only Left.

Mixed Strategies

		Player B	
		L, π_L	$R, 1-\pi_L$
Player A	U, π_U	(1,2)	(0,4)
	$D, 1-\pi_U$	(0,5)	(3,2)

If $2\pi_U + 5(1-\pi_U) < 4\pi_U + 2(1-\pi_U)$ **then B would play only Right. But there are no Nash equilibria in which B plays only Right.**

Mixed Strategies

		Player B	
		L, π_L	R, 1-π_L
Player A	U, π_U	(1,2)	(0,4)
	D, 1-π_U	(0,5)	(3,2)

So for there to exist a Nash equilibrium, B must be indifferent between playing Left or Right; i.e. $2\pi_U + 5(1-\pi_U) = 4\pi_U + 2(1-\pi_U)$

Mixed Strategies

Player B

		L, π_L	$R, 1-\pi_L$
Player A	U, π_U	(1,2)	(0,4)
	$D, 1-\pi_U$	(0,5)	(3,2)

So for there to exist a Nash equilibrium, B must be indifferent between playing Left or Right; i.e. $2\pi_U + 5(1-\pi_U) = 4\pi_U + 2(1-\pi_U)$

$$\Rightarrow \quad \pi_U = 3/5.$$

Mixed Strategies

		Player B	
		L, π_L	R, $1-\pi_L$
Player A	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

So for there to exist a Nash equilibrium, B must be indifferent between playing Left or Right; i.e. $2\pi_U + 5(1-\pi_U) = 4\pi_U + 2(1-\pi_U)$

$$\Rightarrow \quad \pi_U = 3/5.$$

Mixed Strategies

Player B

		L, π_L	$R, 1-\pi_L$
Player A	$U, \frac{3}{5}$	(1,2)	(0,4)
	$D, \frac{2}{5}$	(0,5)	(3,2)

Mixed Strategies

Player B

Player A		L, π_L	R, $1-\pi_L$
	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

If A plays Up his expected payoff is

$$1 \times \pi_L + 0 \times (1 - \pi_L) = \pi_L .$$

Mixed Strategies

		Player B	
		L, π_L	R, $1-\pi_L$
Player A	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

If A plays Up his expected payoff is

$$1 \times \pi_L + 0 \times (1 - \pi_L) = \pi_L.$$

If A plays Down his expected payoff is

$$0 \times \pi_L + 3 \times (1 - \pi_L) = 3(1 - \pi_L).$$

Mixed Strategies

		Player B	
		L, π_L	R, 1-π_L
Player A	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

If $\pi_L > 3(1-\pi_L)$ then A would play only Up.

But there are no Nash equilibria in which A plays only Up.

Mixed Strategies

		Player B	
		L, π_L	R, $1-\pi_L$
Player A	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

If $\pi_L < 3(1-\pi_L)$ then A would play only Down. But there are no Nash equilibria in which A plays only Down.

Mixed Strategies

		Player B	
		L, π_L	R, 1-π_L
Player A	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

So for there to exist a Nash equilibrium, A must be indifferent between playing Up or Down; i.e. $\pi_L = 3(1 - \pi_L)$

Mixed Strategies

Player B

		L, π_L	$R, 1-\pi_L$
Player A	$U, \frac{3}{5}$	(1,2)	(0,4)
	$D, \frac{2}{5}$	(0,5)	(3,2)

So for there to exist a Nash equilibrium, A must be indifferent between playing Up or Down; i.e. $\pi_L = 3(1 - \pi_L) \quad \Rightarrow \quad \pi_L = 3/4.$

Mixed Strategies

			Player B	
			L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$		(1,2)	(0,4)
	D, $\frac{2}{5}$		(0,5)	(3,2)

So for there to exist a Nash equilibrium, A must be indifferent between playing Up or Down; i.e. $\pi_L = 3(1 - \pi_L) \quad \Rightarrow \quad \pi_L = 3/4.$

Mixed Strategies

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

So the game's only Nash equilibrium has A playing the mixed strategy (3/5, 2/5) and has B playing the mixed strategy (3/4, 1/4).

Mixed Strategies

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	(1,2) 9/20	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

The payoffs will be (1,2) with probability

$$\frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$$

Mixed Strategies

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	(1,2) 9/20	(0,4) 3/20
	D, $\frac{2}{5}$	(0,5)	(3,2)

The payoffs will be (0,4) with probability

$$\frac{3}{5} \times \frac{1}{4} = \frac{3}{20}$$

Mixed Strategies

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	(1,2) 9/20	(0,4) 3/20
	D, $\frac{2}{5}$	(0,5) 6/20	(3,2)

The payoffs will be (0,5) with probability

$$\frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$$

Mixed Strategies

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	(1,2) 9/20	(0,4) 3/20
	D, $\frac{2}{5}$	(0,5) 6/20	(3,2) 2/20

The payoffs will be (3,2) with probability

$$\frac{2}{5} \times \frac{1}{4} = \frac{2}{20}$$

Mixed Strategies

Player B

Player A		L, $\frac{3}{4}$	R, $\frac{1}{4}$
	U, $\frac{3}{5}$	(1,2) 9/20	(0,4) 3/20
	D, $\frac{2}{5}$	(0,5) 6/20	(3,2) 2/20

Mixed Strategies

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	(1,2) 9/20	(0,4) 3/20
	D, $\frac{2}{5}$	(0,5) 6/20	(3,2) 2/20

A's expected Nash equilibrium payoff is

$$1 \times \frac{9}{20} + 0 \times \frac{3}{20} + 0 \times \frac{6}{20} + 3 \times \frac{2}{20} = \frac{3}{4}.$$

Mixed Strategies

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	(1,2) 9/20	(0,4) 3/20
	D, $\frac{2}{5}$	(0,5) 6/20	(3,2) 2/20

A's expected Nash equilibrium payoff is

$$1\times\frac{9}{20} + 0\times\frac{3}{20} + 0\times\frac{6}{20} + 3\times\frac{2}{20} = \frac{3}{4}.$$

B's expected Nash equilibrium payoff is

$$2\times\frac{9}{20} + 4\times\frac{3}{20} + 5\times\frac{6}{20} + 2\times\frac{2}{20} = \frac{16}{5}.$$

How Many Nash Equilibria?

- A game with a finite number of players, each with a finite number of pure strategies, has at least one Nash equilibrium.
- So if the game has no pure strategy Nash equilibrium then it must have at least one mixed strategy Nash equilibrium.